

# Lecture 11

Monday, 14 September 2009  
10:27 AM

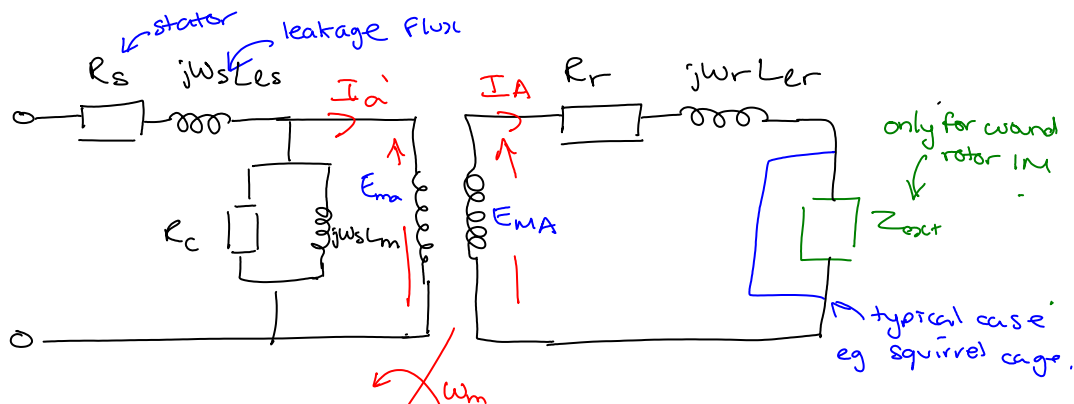
## Definition of slip (Im)

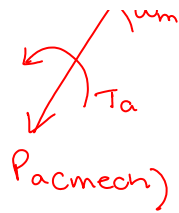
$$\begin{aligned}
 \text{Slip } s &\triangleq \frac{\text{(difference in speed btw rotor \& stator)}}{\text{rotor synchronous speed}} \\
 &= \frac{\omega_{ms} - \omega_m}{\omega_{ms}} \\
 &= \frac{\frac{2}{p} \omega_s - \omega_m}{\frac{2}{p} \omega_s} \\
 s &= \frac{\omega_s - \frac{p}{2} \omega_m}{\omega_s} \\
 &= \frac{\omega_r}{\omega_s} \\
 &= \frac{\text{rotor current frequency}}{\text{stator current frequency}}
 \end{aligned}$$

$$s = \frac{f_r}{f_s}$$

## Single phase equivalent cct

We assume that we are dealing with a balanced 3 $\phi$  Im. Hence it is possible to analyze one phase of the machine and then the results will apply to the other phases by taking into account the phase shifts of  $\pm 120^\circ$





one of three  
balanced phases

## single phase Equivalent circuit

Note that the stator frequency  $\neq$  the rotor frequency  
except for  $\omega_m = 0$ .

Note that  $Z_{ex} = 0$  for rotors with shorted rings  
eg squirrel cage rotors.

The amplitude of the induced EMF in the rotor is  
proportional to the speed difference between the  
flux and the rotor conductors. Clearly, if the  
speed difference is zero, ( $s = 0$ ) then the  
induced EMF is zero.

If the rotor is "locked" in a fixed position, then  
the EMF is determined only from the turns  
ratio ( $s = 1$ ).

$$\therefore \frac{E_{MA}}{E_{ma}} = \frac{\frac{2}{p} \omega_s - \omega_m}{\frac{2}{p} \omega_s} \cdot \frac{N_{re}}{N_{se}}$$

$N_{re}$  = effective rotor turns

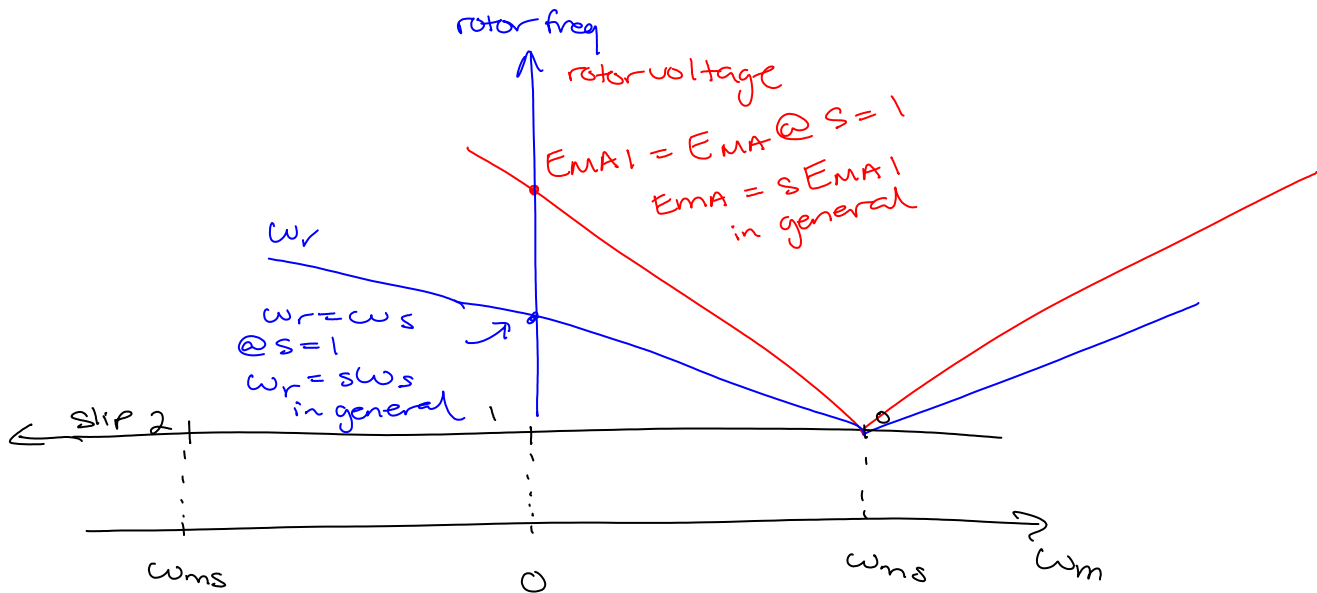
$N_{se}$  = effective stator turns.

$$\begin{aligned} \frac{|E_{MA}|}{|E_{ma}|} &= \frac{\omega_s - \frac{p}{2} \omega_m}{\omega_s} \cdot \frac{N_{re}}{N_{se}} \\ &= \frac{\omega_r}{\omega_s} \cdot \frac{N_{re}}{N_{se}} = s \frac{N_{re}}{N_{se}} \end{aligned}$$

Consider currents:

mmfs must balance:

$$\therefore \frac{|I_A|}{|I_a'|} = \frac{N_{se}}{N_{re}}$$



Examine rotor cct

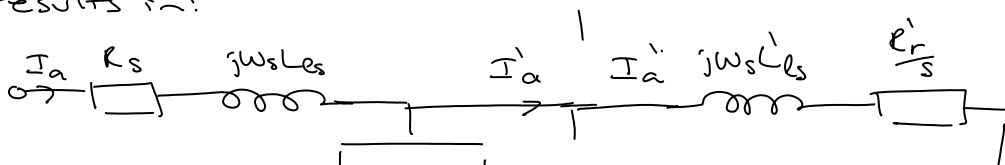
$$I_A = \frac{E_{MA}}{R_r + j\omega_r L_r} \quad (Z_{ex} = 0)$$

$$= \frac{s E_{MA1}}{R_r + j s \omega_s L_r}$$

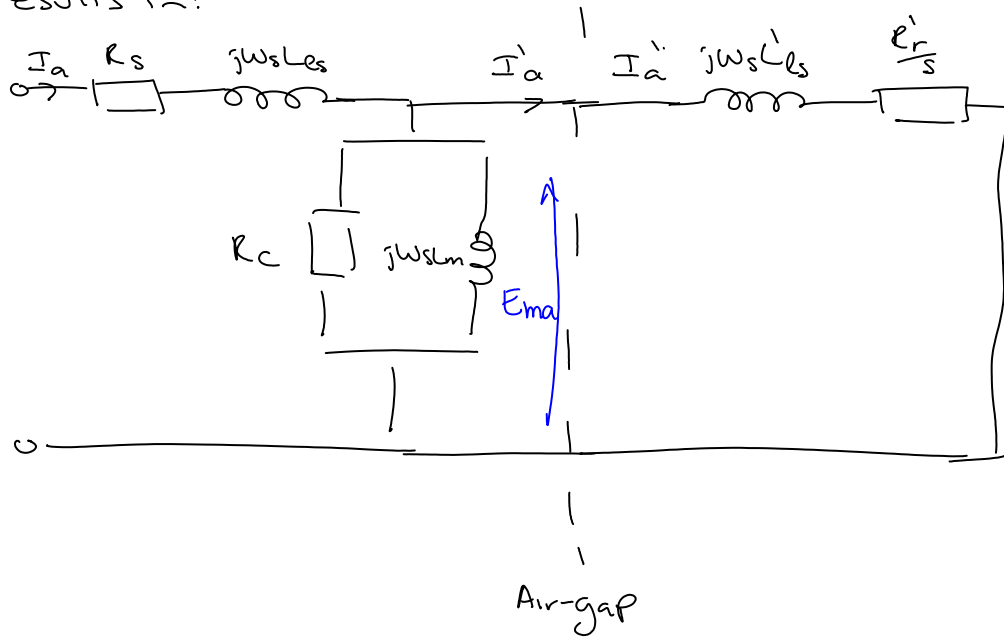
$$= \frac{E_{MA1}}{\frac{R_r}{s} + j \omega_s L_r}$$

This observation allows a changed equivalent cct to be used:

Now, like the transformer we can refer the rotor quantities to the stator and this results in:



results in:



The power transferred across the air-gap is

$$P_{ma} = 3 (I_a')^2 \frac{R_r'}{s}$$

$$= 3 (I_A)^2 \frac{R_r}{s}$$

The rotor winding loss is

$$3 (I_a')^2 R_r' = s P_{ma}$$

and the difference between these must be the mechanical output power.

$$P_{mech} = P_{ma} - 3 (I_a')^2 R_r'$$

$$= P_{ma} - s P_{ma}$$

$$= P_{ma} (1-s)$$

Rotor winding loss =  $s P_{ma}$

$$\boxed{\text{Gross mech O/P power} = (1-s) P_{ma}}$$

$$P_{\text{mech}} = T \omega_n$$

$$\therefore T \omega_n = (1-s) P_{ma}$$

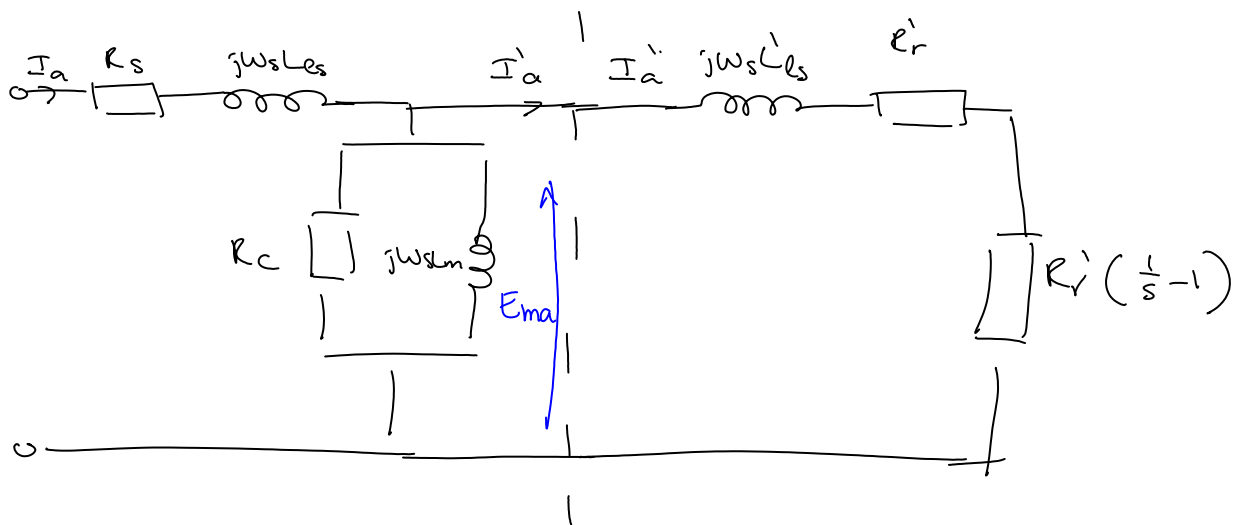
$$\boxed{\therefore \tau = \frac{P_{ma}}{\omega_n}}$$

It is often useful to segregate rotor losses from mechanical O/P power and so the quantity  $R_r'/s$  is separated into

$$\begin{aligned} \frac{R_r'}{s} &= R_r' - R_r' + \frac{R_r'}{s} \\ &= R_r' + R_r' \left( \frac{1}{s} - 1 \right) \\ &= R_r' + R_r' \left( \frac{1-s}{s} \right) \end{aligned}$$

$$\therefore \text{mech O/P power} = P_{ma} - \text{rotor loss}$$

$$= 3 (I_a')^2 R_r' \left( \frac{1-s}{s} \right)$$



To simplify the problem solution we may replace that portion of the equivalent ckt to the left of the air-gap with a Thevenin equivalent.

